THE RELEVANCE LOGIC NEWSLETTER

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Enid Mountin

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Editors: Kenneth W. Collier

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Editorial

It is becoming increasingly difficult, if not impossible, for scholars to keep abreast of developments in their own fields. The proliferation of journals has had the effect of making more results known faster, but in order to read these results, one would have to spend a small fortune in subscriptions and spend inordinate amounts of time doing nothing but reading. And even with this proliferation of journals, there is still an enormous lag-time between submission and publication of articles -- two and even three years being not uncommon.

This newsletter is an attempt to short circuit these difficulties. We have no intention of supplanting already existing journals. Indeed, that would be counter-productive. Instead we hope to supplement those journals. We intend to be the means by which scholars concerned with relevance logic can keep in touch with each other and abreast of developments. To this end, we will publish abstracts of papers, short survey articles, suggested problems, review articles, bibliographic notices, etc. We will not publish long articles or reproduce proofs in great detail as that is the function of the established journals.

To succeed in this, we must be kept informed. We are, then, taking this opportunity to ask our readers to send us notices of their results, copies of their publications, reviews and anything else that you think the other readers would be interested in seeing. Our assumption is that people want to be kept informed, but that implies a willingness to do some informing.

--The Editors

VARIA

It is appropriate to inaugurate this Newsletter by celebrating in print the publication of volume 1 of Alan Ross Anderson and Nuel D. Belnap, Jr.'s Entailment. The Logic of Relevance and Necessity, Princeton (Princeton University Press) 1975. For those readers of the Newsletter who - per impossibile - have not seen it, we reproduce the following vital statistics. The book is 542 + xxxii pages and sells for US $21.00. Its Library of Congress number is 72-14016 and its International Standard Book Number is 0-691-07192-6.

Nuel Belnap is interested in collecting a list of printing (and other) errors. Those who discover any are invited to send him a note informing him of such errors. The Newsletter would also be interested in being informed of such errata; we hope to publish a list in a future issue.

Volume 2 is due out "about a year after" volume 1. Robert K. Meyer's name has been added to volume 2 as a coequal author with Anderson and Belnap.

Bob Wolf, a coeditor of the Newsletter, is preparing a Comprehensive bibliography for volume 2 and would be interested in any information about bibliographical items, especially unpublished ones.

We would be interested in getting copies of any reviews or critical notices of Entailment which any of our readers might prepare. We would in fact be interested in publishing such notices insofar as they include technical or philosophical points which would stimulate discussion or research.

Two other monographs, listed in the bibliography, published in 1975 should be of special interest to our readers, even though they are not about relevant logics. The first is Ernest W. Adams' The Logic of Conditionals which presents a non-truth-functional logic based on the calculus of probability. The second is Arthur W. Burks's Cause, Chance and Reason which presents a non-truth-functional causal logic. Both monographs include discussions of applications of such logics to philosophical issues.

Richard Routley's brief outline of research activities in Canberra, elsewhere in this issue, deserves some further elaboration:

1) Richard and Bob Meyer are preparing a monograph for publication in the Department of Philosophy, The Research School of Social Sciences, The Australian National University monograph series, entitled: Relevance Logics and Their Rivals. The monograph greatly extends the number of propositional systems treated with the Routley-Meyer-Fine ternary relation semantics, among them many systems with non-classical negations, systems with multiple modal connectives (including relevant tense and deontic logics) and some of the Lewis systems. The semantics also links up with the Routley-Meyer universal semantics which is the subject of a forthcoming article by Routley in the Journal of Philosophical Logic and an article by Meyer and Routley forthcoming in The Monist. Two features of the monograph seem likely to generate discussion: 1) an argument by Richard and Valerie Routley for the philosophical adequacy of relevant logics weaker than E as "better" formalizations of entailment, and 2) an argument for the philosophical superiority of S5 modal structures over S4 modalities in a relevant logic.

2) Richard and Valerie Routley are preparing for publication a monography entitled: Beyond the Possible. The monograph will include a philosophically oriented defense of the Routley-Meyer semantics and apply the semantics to several current topics of philosophical interest, including the construction of relevant Carnapian measure functions for probability.

3) Michael Dunn, who is in Canberra until July 1, 1976, is working on the semantics of quantified modal logics. He is completing a paper on the "Completeness of RMQ". Hopefully we will be able to report in more detail on his work in a future issue. Dunn has recently completed a series of papers on relevant logic (before he left for Canberra). Among them is a paper on a Kripke-style semantics for RM, an intuitive semantics for E_{fdf} (due to be published in Philosophical Studies) and a semantical analysis of the systems between RM and TV and the series between R and RM (due to be published in the Notre Dame Journal of Formal Logic).

4) Nuel Belnap is in Canberra until April 1, 1976. He is working with Meyer on the preparation of the manuscript for volume 2 of Entailment and deciding with Bob Meyer whether or not there will be a third volume also. Belnap has recently developed the intuitive semantics for E_{fdf} first proposed by Dunn, and established some extremely fruiful connections between the semantics and Dana Scott's theory of approximation lattices. One of the papers arising out of the work is due to be published in Modern Uses of Multiple-valued Logic, edited by J. Michael Dunn and George Epstein, Dordrecht (D. Reidel Publishing Company) 1976.

5) Bob Meyer, who is in Canberra until about December 1, 1976, has been extremely active, working on the sections of Entailment, volume 2,

devoted to the semantics of entailment. He has also been working on a long (100+ pages) paper on first-order relevant Peano arithmetic and a paper extending his coherence methods to prove the admissibility of γ for a wide range of higher-order relevant logics; we hope to include a report on both areas of research in our next issue.

The system of first-order relevant Peano arithmetic (called R#) is provably absolutely consistent in that there is a simple model hwich shows that "0 = 1" is not provable in R#. It is however to prove the negation consistency of R# within R#. Meyer's results are, we feel, the deepest yet in the relevant logic field and illustrate the same phenomenon in arithmetic as in the study of propositional logics: concepts which are collapsed together in the classical case have to be separated off from each other and treated individually in the relevant case.

Meyer has also recently finished a paper answering two questions posed in Entailment in the negative: one given on page 97 and one of page 94. The paper besides its negative answer to the original open questions includes an interesting technical discussion of the algebra of NR and a valuable discussion of the philosophical importance of Hallden-completness and its cognate principles.

6) We can only echo Routley's regret that this fine group will be broken up during the current year. The current address for all of the above is:

Department of Philosophy
The Research School of Social Sciences
The Australian National University
Box 4, P. O.
Canberra, A. C. T. 2600
Australia

Jerzy Perzanowski tells us that the Department of Logic of Jagiellonian University is conducting a year-long seminar on relevant logics and that at least one participant in the seminar is planning to write a dissertation on the subject. The Department of Logic is interested in obtaining copies of papers by logicians for their files and corresponding with other logicians. The Department also publishes the Reports on Mathematical Logic which you might keep in mind as a possible outlet for papers. Professor Perzanowski's address is:

Department of Logic
Jagiellonian University
ul. Grodzka 52
31-044 Kraków
Poland.

Charles F. Kielkopf has recently finished the first draft of a monograph, entitled Formal Sentential Entailment, which is a defense of the philosophical adequacy of E as the correct logic of entailment. The monograph includes as well as a discussion of many of the current entailment logics a discussion of the intuitive meaning of the Routley-Meyer semantics for E.

Professor Kielkopf informs us that he has a few copies of the monograph available to those interested. His address is:

Department of Philosophy
The Ohio State University
Columbus, Ohio 43210

We are pleased to be able to publish in this inaugural issue of the Newsletter two pieces by Willian T. Parry of SUNY at Buffalo which discuss his system of Analytic Implication. The first, an abstract of a 1963 paper, is the item mentioned on page 430 of volume 1 of Entailment; the second is the original of a paper abstracted in JSL in 1972.

The Story of NR and E

In [6] we find the following reasoning: the calculus R arises from adding considerations of relevance to the usual logical motivation. If entailments are necessary relevant implications, then the calculus that arises from the following definition

$A \Rightarrow B =_{df} \Box(A \rightarrow B)$, where '$\rightarrow$' is the implication relation of R,

ought to be the calculus of entailment. This calculus was dubbed 'NR'. This name has been supplanted by '$R^{\Box}$' in a friendly ammendment to Meyer's paper. If this line of reasoning is correct and the reasoning that motivated E is correct, then it ought to be the case that $\vdash_{R^{\Box}} A \Rightarrow B$ iff $\vdash_{E} A \rightarrow B$. Unable to prove that this is so, Meyer conjectured that it is so.

From 1968 until 1971 virtually everyone thought that Meyer's conjecture was correct. This became such an article of faith that Anderson and Belnap wrote "...it just must be true (Method of Authority); but thought it is difficult to speak confidently, we predict that if in fact it is found that $R^{\Box}$ and E diverge, then we shall, with many a bitter tear, abandon E." But in 1972, G.E. Minc proved in [7] that

$(A \Rightarrow (B \Rightarrow C)) \& (B \Rightarrow (A \vee C)) \Rightarrow (B \Rightarrow C)$

is a theorem of $R^{\Box}$, and conjectured (though he could not prove) that the corresponding formula is not a theorem of E.

In 1973 the ax fell. In [5] L.L. Maksimova constructed a semantics for E, and using those semantics, she proved that E rejects

$(A \rightarrow (B \rightarrow C)) \& (B \rightarrow (A\ C)) \rightarrow (B \rightarrow \check{C})$.

$R^{\Box}$ is not E! (This result appeared between the writting of Anderson and Belnap above and the publication of [1]. In [1] they report Maksimova's result and postpone the decision whether to laugh or cry until volume 2.) Chidgey [2], has found an 8X8 matrix and Kielkopf, [4], has found a 16X16

Short Report of 1975 Research at the Research School of Social Science, The Australian National University

In logic there has been much activity owing to the exceptionally strong group of logicians temporarily in the Department. There have been regular reading and research sessions on formal algebra and combinatory logic and on formal languages and automata theory, and a logic work-in-progress seminar has been firmly established. There have as well been many occasional seminars. Research in logic has centered so far on the following topics, the first of which unifies many of the other investigations: extending logic to take account of further forms of discourse; semantics for formal and natural languages and extensional reduction; applications of type theory and significance logic to analysis of language and philosophical problems; the logics of positivism and of evidence, in particular within the frameworks of intensional significance logic; semantical analyses of quantified relevant logics; and the formalisation of relevant arithmetic and set theory and applications of relevant logic to the foundations of mathematics and to classical consistency questions. Unfortunately this splendid group in logic will be largely disbanded next year.

--Richard Routley

matrix which duplicate Maksimova's proof.

The outstanding question, then, is whether or not the calculus of entailment ought to accept the Minc-Maksimova formula. Aside from an implied promise to take up the matter in volume 2 of Entailment, the only logician I am aware of who discusses the question is Kielkopf. In [3] he considers an instance of the M(inc)-M(aksimova) formula:

$$(D\to D\to .D\to D)\&(D\to(D\to D)\vee D)\to .D\to D,$$

and argues that since the first conjunct of the antecedent is an instance of the consequent and the second conjunct is an instance if an axiom of E not necessary in deriving the consequent, this instance of the M-M formula ought not be a valid entailment and thus the M-M formula ought to be rejected. He also argues that R^{α} puts the cart before the horse. In effect he claims that we should understand necessity in terms of valid entailments rather than entailments in terms of statements of necessity. (Note that if this were correct, then E would be the favored system even if it and R^{α} did coincide.) And this is where the matter rests at the present time.

Kenneth W. Collier

Southern Illinois University at Edwardsville

[1] Anderson, Alan R. and Belnap, Jr. Entailment: The Logic of Relevance and Necessity, vol. 1. Princeton (Princeton University Press), 1975.

[2] Chidgey, John R. On Entailment: A Consideration of Some of the Formal Properties of the Anderson and Belnap System E of Entailment and Related Systems. Victoria University of Manchester Doctoral Dissertation, 1974.

[3] Kielkopf, Charles F. Formal Sentential Entailment, unpublished dittograph, 1974.

[4] Kielkopf, Charles F. "Two E Matrices". Unpublished typescript, 1974.

[5] Maksimova, Larisa L. "A Semantics for the Calculus E of Entailment", Bulletin of the Section of Logic, Polish Academy of Sciences, Institute of Philosophy and Sociology, vol. 2, pp. 18-21.

[6] Meyer, Robert K. "Entailment and Relevant Implication", Logique et Analyse, n.s. vol. 11, pp. 472-479.

[7] Minc, G.E. "Teoréma ob ustranimosti sečénia dlá releventnyh logik", Isslépovaniá po konstruktivnoj matematike i matematičeskoj logike, ed. J.V. Matijasevic and O.A. Slisenko, Leningrad (Isdatel'stva Nauka), pp. 90-97. English translation ("Cut-Elimination Theorem in Relevant Logic") to appear in The Journal of Soviet Mathematics.

ABSTRACT: Entailment: Analytic Implication vs. the Entailment of Anderson & Belnap

"p entails q" means the same as "q is deducible from p (by some valid mode of inference)." Lewis's strict implication is a standard of comparison for theories of entailment. Theorists of entailment are divided on analysis of the paradoxes of strict implication. One of these (Paradox 1a): (p and not p) strictly implies q. Argument I for this" "If p . -q entails r, the p . -r entails q; now p . -q entails p; therefore p . -p entails q."

Two systems of entailment that reject the first premise and conclusion of this argument, accepting the 2nd premise, are Analytic Implication (AI) of W. T. Parry, and Entailment (E) of A. R. Anderson and N. D. Belnap, Jr.

(AI) If a system contains the proposition "2 points dtermine a straight line," must it necessarily contain the theorem "Either 2 points determine a straight line or the moon is made of green cheese"? No, the latter is not a legitimate theorem contains - undefined or defined - the terms "moon" and "green cheese." Hence "p entails p or q" is not universally valid. In general, no entailment formula is valid for AI which contains a free variable in the consequent that does not appear in the antecedent (Proscriptive Principle of AI, or principle of analytic relevance).

(E) Anderson-Belnap's system E first appeared in J. Sym. Log. (1958) 457, and is a slight modification of Ackermann's "strenge

Implikation" (rigorous implication) JSL (1956). E satisfies a "minimal requirement for relevance": if A entails B (in propositional logic), A and B share a propositional variable. (But E accepts "p entails p v q".) Also, if A entails that B entails C, A must contain an entailment.

Argument II for Paradox 1a: (-p . p) entails (-p . (p v q)); (-p . p v q) entails q; Entailment is transitive; therefore (-p . p) entails q." AI and E accept the transitivity of entailment, and reject the paradox. Both accept some "principle of the factor" that would go from "p entails p v q" to the first premise of Argument II. AI, by its Proscriptive Principle, rejects "p entails p v q" and the first premise; it accepts the second premise (disjunctive syllogism). A & B accept "p entails p v q" and the first premise, and reject the disjunctive syllogism.

A & B say that p v q is contained in p "in the sense in which the concept Sibling (... defined as Brother-or-Sister) is contained in the concept Brother." They don't discuss the case of a totally irrelevant q.

When A & B discuss a concrete example of a disjunctive syllogism, they find it to be valid, but hold that the "or" premise is not merely truth-functional. The sriter submits that, taking a disjunction acknowledge by A & B to be truth-functional ("Either Napoleon was born in Corsica or the number of the beast is perfect"), the disjunctive syllogism is a valid entailment. But A & B have to reject the

in addition, the following, theorems of above set:

0.2 $p \to pp$ 0.31 $--p \to p$

0.5 $(p \to q)(q \to r) \to .\ p \to r$ 0.7 $(p \to r)(q \to s) \to .\ pq \to rs$

0.73 $(p \to r)(q \to s) \to .\ p \vee q \to r \vee s.$

An unpublished paper ('57)* added 2 (independent?) postulates:

(2d) $-\Diamond -p\ .\ p \to q.\ \to -\Diamond -q$ (3b) $-(p \supset q) \to -(p \to q).$**

"$\Diamond p$" was defined "$-(p \to -p)$"; a more general conception makes "$\Diamond p$" equivalent to "$-(\exists q)(q \supset q. \to -p)$".

Matrices satisfying AI. 1', 1 are the designated values.

pq	0'	1'	0	1	-p
0'	0'	0'	0	0	1
1'	0'	1'	0	1	0'
0	0	0	0	0	1
1	0	1	0	1	0

→	0'	1'	0	1
0'	1'	1'	0	0
1'	0'	1'	0	0
0	1	1	1	1
1	0	1	0	1

No →-formula is satisfied which has a variable not occurring in antecedent. (Substitute 0 for such variable(s), 0' for variables in antecedent.

SYSTEM E of Anderson & Belnap (Philosophical Studies 1962).

"$A \to B$" for "A entails B"

Undefined: →; -, &, v

Axiom schemata:

1 $(A \to A) \to B. \to B$ 2 $A \to B \to : B \to C \to .\ A \to C$

3 $(A \to .A \to B) \to .A \to B$ 4 $A \ \& \ B \to A$

5 $A \ \& \ B \to B$ 6 $(A \to B) \ \& \ (A \to C) \to .A \to B \ \& \ C$

7 $\underline{N}A \ \& \ \underline{N}B \to \underline{N}(A \ \& \ B)$ 8 $A \to A \vee B$

[$\underline{N}A =_{df} (A \to A) \to A.$]

disjunctive syllogism, else E contains Paradox 1a.

References: Strict Implication: Lewis & Langford Symbolic Logic, espc. chap. 8.

On system E: Anderson & Belnap, "Tautological Entailments," Philosophical Studies, vol. 12, Jan-Feb 1962. (& "Enthymemes," A & B, Jr. of Phil., vol. 58 (1961), 713-23.).

On Analytic Implication: Summaries of Theses ... 1932, Harvard U (1933), pp. 332-5; Ergebnisse eines math. Kolloquiums (1933), 5-6. Only full account in dissertation, Implication, esp. (critique) pp. 108-36, 229-39; (system of AI) 168-201, 202-28, 240-2.

ANALYTIC IMPLICATION. System AI,

Undefined: Analytic implication $p \to q$; truth-functions $-p$, pq.

Defined as usual: Other truth-functions; Analytic equivalence $p \rightleftharpoons q$,

Postulate set equivalent to original 1931 set:

(1a) $pq \to qp$ (1b) $p \to --p$

(1c) $p \cdot (q \vee r). \to pq \vee pr$ (1d) $p \vee (q \cdot -q) \to p$

(2a) $p \to qr. \to . p \to q$ (2b) $(p \to q)(p \to r) \to . p \to qr$

(2c) $(p \to r)(q \to r) \to . p \vee q \to r$ (3a) $p \to q. \to . p \supset q$

(4a) $f(p) \to . p \to p$ (4b) $p \rightleftharpoons q \cdot f(p). \to f(q).$

[f(p) stands for any wff containing p.]

Rules of Inference: Substitution for variables, Replacement for definitions, Modus ponens for $P \to Q$, Adjunction (from P, Q to PQ). 1a, 1b, 1c, 1d, 2a, 3a, 4a, 4b were in original postulate set;

9 $B \rightarrow A \vee B$ 10 $(A \rightarrow C) \ \& \ (B \rightarrow C) \rightarrow .A \vee B -$

11 $A \ \& \ (B \vee C) \rightarrow (A \ \& \ B) \vee C$ 12 $A \rightarrow -A. \rightarrow -A$

13 $A \rightarrow -B \rightarrow .B \rightarrow -A$ 14 $--A \rightarrow A.$

Rules: Modus ponens: If $A \rightarrow B$ is asserted, then from A to infer B.

Adjunction: From A and B to infer $A \ \& \ B$.

Set of 8-element matrices shows that $A \rightarrow B$ is provable in B only if A and B share a variable; also that "$A \rightarrow .A \rightarrow A$" fails, and "$-(A \rightarrow A) \rightarrow .A \rightarrow A$" fails.

HOLD in BOTH AI & E:	HOLD in AI, not E:	In E, not AI:
AI 1a, 1b, 1c, 2a, 2b, 2c, 3a; 0.2, 0.31, 0.5, 0.7, 0.73; 3b.	AI 1d, 4a $-p \ . \ p \vee q. \rightarrow q$ $p \ . \ p \supset q. \rightarrow q$ $p-pq \rightarrow -q$	E 2, 8, 9, 13; Converse of 10; $A \rightarrow B \rightarrow :C \rightarrow A \rightarrow .C \rightarrow B.$
E 1, 3, 4 - 7, 10 - 12, 14		

NEITHER AI not E:

$p-p \rightarrow q$

$q \rightarrow p \vee -p$

$p \rightarrow pq \vee p-q.$

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* "The Logic of C. I. Lewis," in Phil. of C. I. Lewis (1968).

** J. M. Dunn (1972) showed that 3b is independent.

William T. Parry

March 1963

Comparison of Entailment Theories

It is well known that use of the term 'entailment' as a technical term of logic goes back to Moore's paper on "External and Internal Relations" (Proceedings of the Aristotelian Society, 1920). He proposed to use 'p entails q' to express the converse of the relation taht holds between propositions q and p when q "follows from or is deducible from" p. He said that 'follows from' is taken "in the sense in which the conclusion of a syllogism in Barbara follows from the premises, taken as one conjunctive proposition; or in which the proposition 'This is coloured' follows from 'This is red.'" In the original paper, he said (p. 58) that he doubtless had the same relation in mind as C. I. Lewis in his studies of Strict Implication. But when Moore reprinted his paper in Philosophical Studies (1922), he omitted all reference to Lewis. Nevertheless, Lewis's system of Strict Implication is of course a form of entailment-system. And, because of his painstaking work in formulating, developing, and defending a system--or systems-- of entailment, and because also of the inevitability of this particular sort of concept of entailment, Lewis's studies have been the principal point of reference for entailment theories.

As Lewis says, he is primarily interested in that meaning of 'p implies q' which is synonomous with 'q is deducible from p' [SL, p. 122]. He further clarifies what 'deducible' means here. It means, he says, "(1) 'deducible by some valid mode of inference'," not

> "(2) 'deducible in accordance with some principle of inference which has been assumed or already proved.' The first [he goes on] is the only meaning which ever occurs outside of logic itself; in mathematics

ordinarily mean by 'deducibility') of course is transitive. So, in the sense in which Lewis and we use the term, 'deducibility' is necessarily transitive.

Here it is instructive to consider the viewpoint of T. J. Smiley, whose "Entailment and Deducibility" (Proceedings, Aristotelian Society, 1959, 233-54) is still the best systematic survey of theories of entailment I have seen. He writes:

> ". . . the whole point of logic as an instrument, and the way in which it brings us new knowledge, lies in the contrast between the transitivity of 'entails' and the non-transitivity of 'obviously entails', and all this is lost if transitivity cannot be relied on." (p. 242)

I certainly agree. Yet we must also harken to his statement on the same page:

> "The need for an unrestrictedly transitive entailment-relation for serious logical work in no reason at all against accepting a relation which is not unrestrictedly transitive as being a satisfactory reconstruction of an intuitive idea of entailment".

I can't agree that this need is "no reason at all"; but I allow that it should not preclude an consideration of formulation that captures some intuitive idea with its own raison d'etre. Only, as a logician, I insist on having one or more transitive entailment-relations at hand, as does Smiley himself.

An important formal difference exists between theories of entailment which introduce entailment as a modal relation in the object language, and those which have entailment only in the metalanguage. For some

> or politics or what not, to say that something is deducible means that it may be inferred by some valid mode of reasoning (all such valid modes being taken for granted). But in logic itself, the propositions of the system have a double use: they are used as premises from which conclusions are drawn, and as principles of inference in accord with which conclusions are drawn. Therefore in logic, 'deducible' may have [either of the two meanings]. . . . let us call this second meaning of the word 'logistic deducibility.'" [SL (1932, 1959), pp. 252 f; all stress as in original.]

I think it is necessary to make a distinction of two senses of 'deducibility' also within the application of logic to non-logical material. Logicians are no longer inclined to take for granted the ready availability of all valid modes of reasonong. We might speak of the immedidately deducible. what can be inferred at the next step by means of any principle stated in some extensive canon such as PM, for example, or any principle derivable by an available decision procedure, e.g., any truth-functional tautology. This is distinguished from the ultimately deducible, which is arrived at by any finite series of immediately deducible steps. Within a system of logic, there is an analogous distinction between an immediate logistic deducibility on the one hand, the content of which however is often enlarged as the system is developed, and ultimate logistic deducibility on the other hand. The immediate deducibility (logistic or or not) is not transitive, but the ultimate deducibility (which is what we

exponents of theories of the latter sort, a rejection or distrust of modality in the object-language is involved. In Smiley's 1959 paper, various systems of entailment are presented, using the assertion sign as metalogical sign, read 'A entails B', or A_1 and . . . and A_n entail $\underline{B}$'. He says that, although the entailment-sign "could be treated as a connective (if the whole discussion . . . were to be formalized along the lines of a modal logic) it would have to be a connective with no fixed number of argument-places. Its customary treatment as a binary connective in systems of strict implication confuses 'A & B entails C' with 'A and B entail C' (in which 'and' is no more a connective than it is in, e.g., 'A and B entail each other')." (235, n. 2)

I see no confusion in the binary entailment-connective of a system like Analytic Implication, in which we may introduce or eliminate conjunction at will, and, if we had an n-place entailment-connective, 'A & B entails C' would be interdeducible and interchangeable with 'A and B together entail C'. However, I acknowledge that there would be some advantage for Analytic Implication in using an n-place entailment-connective. Analytic Implication has--analogous to the T-principles of Lewis's S1--many principles which require in the antecedent a conjunct of the form 'p entails p' or perhaps 'p or not-p'. Perhaps it would seem less odd if the tautological premise need not be explicitly conjoined together with the other premisses in order to entail the consequent. It is sufficient that they be asserted in the same system in order for them to have a joint consequence. There is, we might say, a kind of loose conjunction asserted between any two propositions of the same system.

We wish to mention but not pursue the distinction between a purely

This conclusion we call Paradox 1.

Lewis--and most logicians, apparently--accept the premisses and hence the paradox. (Cf. Lewis, SL, App. III, p. 512.) So did P. Vredenduin (Jr. Sym. Logic, 1939), who developed a slightly stricter implication, holding that there might be "other kinds of impossible propositions" than those from which a proposition of the form p & -p can be derived.

E. J. Nelson (1930) accepted TTra, but rejected Simplification on intuitive grounds, and escaped the paradox. The third family, which accepts simplification and rejects the paradox, hence must reject TTra, contains most of the rival of Strict Implication, though not all of them proclaim their rejection of TTra.

Returning for a moment to Nelson: he held that if the conjunction of p and q entails r, it should be by a "joint force" of p and q, and not as a mere aggregate. He tried to rule out cases where a formula may degenerate to an instance of "pq entails p". Thus he imposed a non-equivalence condition on the variables for his formulation of the transitivity of entailment, to present it from degenerating to an instance of Simplification. However he does not avoid all such instances: one can prove "p -(p -p) E p" in his system. (See "Logic of C. I. Lewis," note 115.)

The best policy for a system rejecting Simplification, I think, is that recommended by some critics and worked out by Smiley in his dissertation, as reported in the article cited: One need not exclude instances of simplification reached by degeneration of other formulas. Smiley's theory "requires not that there should be no cogent way of reaching the conclusion without using all the premisses, but only that there should be some cogent way of reaching it

formal or structural interpretation of entailment on the one hand, and on the other hand an intensional interpretation for which such a statement as "That this is red entails that this is colored" holds directly, without need for a premise explicitly connecting 'red' and 'colored'. The examples in both Moore and Lewis show clearly that they gave their relations the intensional interpretation, even though the discussion in some passages of Symbolic Logic suggests he was thinking at the moment in structural terms. (Cf. Parry, "Logic of C. I. Lewis", in Schilpp volume, §vii, p. 140.) We conjecture that while specific systems of Strict or Analytic Implication seem to be interpretable either structurally or intensionally, a careful analysis of just what one takes to be valid for an intensional implication (perhaps by way of a reduction of it to a structural implication) might lead one to adopt distinct strict formal systems for the two interpretations.

We turn to the division of systems of entailment on the basis of their acceptance or rejection of principles of propositional logic, especially first-degree entailments or conditionals relating 1st-degree entailments. These divisions are made most neatly on the basis of reaction to arguments for paradoxes of strict implication.

Systems of entailment divide into three families by the reaction to a simple argument for a paradox of strict implication.

By Triadic Transposition, abbreviated 'TTra', (often called 'Antilogism'):

If p & -q entails r, then p & -r entails q

By Simplification: p & -q entails p

Substituting in TTra, p for r,

∴ p & -p entails q.

with all the premisses used; thus it admits $(A \supset A)$ & $(A \supset A) \vdash A \supset A$ precisely because it exemplifies the acceptable principle $(A \supset B)$ & $(B \supset C) \vdash A \supset C$, though it does also exemplify the rejected principle A & B $\vdash$ A." (p. 249.)

We might mention R. B. Angell's "A Propositional Logic with Subjunctive Conditionals" (Jr. Sym. Logic, 1962, 327-43). This does not purport to be a theory of entailment, but of subjunctive conditionals. Simplification is rejected, but at least one theorem (*23) can become an instance of Simplification. Like Nelson, Angell has formulas whose analogues do not hold for Strict Implication: e.g. A_{10} $(p \rightarrow q) \nrightarrow -(p \rightarrow -q)$.

The idea of entailment or conditional without Simplification evidently has an intuitive appeal, and keeps recurring. It appears in writers in the Proceedings after Smiley, so do not refer to him; e.g. R. W. Ashby (1962-63). But Nelson himself came to doubt its correctness (1933, 1936); Smiley does not regard his own version of a Nelson-like system as the best account of entailment; and Angell does not present his system without Simplification as a system of entailment.

Now the third family, which accepts Simplification but rejects TTra and the paradox, includes my Analytic Implication (1933), and the systems of Hallden (1948), Ackermann (1956), Anderson and Belnap (1958) and von Wright-Geach.

A second argument for Paradox 1 differentiates among these. This argument is adapted from Smiley. (Essentailly the same argument was given by Grice the day before this paper was read in a simpler form, but the more complicated form after Smiley suits my purposes a little better.)

Add(ition): p entails p v q (1)

Factor: If $\vdash$(P entails Q), then $\vdash$(P & R entails Q & R) (2)

We generalize to a Proscriptive Principle, which Analytic Implication satisfies: No formula with Analytic Implication as main relation holds universally if it has a free variable in the consequent that is not in the antecedent.

This principle is much stronger than the relevance principle of Anderson and Belnap (which, I understand, it suggested to them). We can provide doctored-up forms of the missing principles; for example, p & (q v -q) → p v q.

We must return to the other members of our third family. S. Hallden's system "S0" [zero] (Theoria, 1948), made from Lewis's S1 without the definition of strict implication, is not in the Lewis family, and is in fact a subsystem of Analytic Implication. He explicitly rejects Addition as "highly paradoxical" for entailment. Hallden was not aware of Analytic Implication, and ascribes his distinction of entailment from Strict Implication to Duncan-Jones (1935).

Ackermann accepts Add, Factor, and PHS (transitivity); he rejects Disjunctive Syllogism as an entailment--somewhat anomously, since he has it as a rule. Anderson-Belnap's system E follows Ackermann, except that they unequivocally reject DS even as a rule.

The von Wright-Geach system rejects PHS, but accepts the other principles involved here. The same holds for Smiley's system of this type (I, §2), in which (1st-degree) entailments must reduce to a substitution-instance of a tautology with antecedent and consequent contingent. In a variant of this idea, in which entailments hold only between contingent propositions, (I, §3) Smiley finds the chain breaking with Factor rather than PHS.

The first-degree entailments of the propositional calculus of AI are the formulas "P → Q" such that "P ⊃ Q" is a truth-functional tautology, and Q

By (2), since (1):	p & -p entails (p v q) & -p	(3)
D(isjunctive) S(yllogism):	(p v q) & -p entails q	(4)
P(ure) H(ypothetical) S(yllogism)	If P entails Q and Q entails R then P entails R	
By PHS, since (3) & (4):	p & -p entails q	(Paradox 1 again)

Analytic Implication rejects Addition, accepts DS and PHS. This is no ad hoc rejection. Indeed, Analytic Implication started as an elaboration of the view H. M Sheffer expressed in class, that a conjunction entails its conjuncts, but a proposition doesn't necessarily entail its disjunction with any other proposition.

Our conception of deducibility may be clarified thus: Q is deducible from P if, in any system in which P is asserted, the assertion of Q is justifiable, assuming a reasonably complete logic. Now, we ask, if a system of Euclidean geometry contains the assertion that two points determine a straight line, are we justified in asserting in this system: "Either two points determine a straight line or some mice like cheese"? No, this strange disjunction is not a legitimate assertion in a system of Euclidean geometry, for the simple reason that no such system contains the terms "mice" or "cheese", nor can one define by geometric concepts a type of cheese any self-respecting mouse would nibble at. We conclude, then, that it is not true that the statement that either two points determine a straight line or some mice like cheese is deducible from the statement that two points determine a straight line. So the statement that either p or q is not always deducible from the statement that p; putting it the other way around, we find that p does not always entail p or q.

contains no variable not contained in P.

System E contains a restriction not contained in AI, that only entailments entail entailments, so that "$A \rightarrow (A \rightarrow A)$" is rejected. It is acknowledged that it would be a natural modification of AI to adopt a similar principle, or at least to require that the entailment sign cannot appear in the consequent of an entailment unless it appears in the antecedent.

E, however, contains the principle that $A \ \& \ -B \rightarrow -(A \rightarrow B)$, which derives a non-entailment from a truth-function. This (the principle of the counter-example) is a very plausible and tempting principle, which, in my paper, "The Logic of C. I. Lewis," I proposed to add to Analytic Implication. It doesn't conform to the Anderson-Belnap contention that the truth or falsity of an entailment is independent of any truth-functional statement. From my point of view it is perhaps objectionable as introducing the concept of entailment without antecedent, and perhaps I should have this rather in such a form as $p \ \& \ -q \ \& \ \pm(r \rightarrow s) \rightarrow -(p \rightarrow q)$.

It seems to me now, then, that there are two tenable alternatives for AI with regard to introduction of an entailment into the consequent of an entailment when no entailment was in the antecedent. Either (1) entailment should be consistently treated like truth-functions, which may appear in the consequent regardless of appearance in the antecedent; or (2) entailment should be treated like empirical concepts, which may not appear in the consequent of an entailment unless they (or empirical concepts from which they may be derived) are in the antecedent. I can see no reason, from the stanpoint of AI, why it should make any difference whether an entailment or a non-entailment is involved, any more than it matters for

the availability of an empirical concept whether it has been affirmed or denied of something. In the first case, the assumption of the original system of AI--$f(p) \rightarrow .p \rightarrow p$--should stand, and be supplemented by the above-mentioned principle of the counter-example in its simple form (as suggested in "The Logic of C. I. Lewis"). In the second case, each of these principles should appear only "doctored up" to require the presence of an entailment in the antecedent. Here again Smiley's idea of an n-place entailment connective would enable this "doctoring up" to occur in a simpler and more natural way, not requiring the explicit conjunction of the propositions involved.

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Note: This is the paper as presented in December 1971, with practically no change except in the last paragraph. An abstract of this paper appeared in JSL, vol. 37 (1972), 441 f.

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